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Bisplitting an Arbitrary N-Qubit State with a Class of Asymmetric Three-Qubit W States

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Abstract We propose a tripartite scheme for bisplitting an arbitrary single-qubit quantum information (QI) by using a class of asymmetric three-qubit W state as quantum channels. In the scheme, the sender Alice first performs a Bell-state measurement on her two qubits and then publishes her measurement result via a public classical channel. Based on Alice's message, if the two receivers Bob and Charlie collaborate together, they can retrieve the QI, i.e., they can deterministically recover the QI in one receiver's site by first performing a two-qubit unitary operation and then executing a single-qubit unitary operation. Afterwards, we sketch the extension of the single-qubit QI splitting to an arbitrary N-qubit QI splitting.

Keywords Quantum information splitting · Asymmetric three-qubit W state · Arbitrary single-qubit state · Arbitrary N-qubit state

1 Introduction

Quantum secret sharing (QSS) was first proposed by Hillery, Bŭzek and Berthiaume (HBB) [1] in 1999 by utilizing three-particle and four-particle Greenberger-Horne-Zeilinger (GHZ) states. It is in essence a generalization of classical secret sharing [2, 3] in quantum scenario. It comprises both secret classical-message quantum sharing and secret quantum

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state splitting (SQSS). Incidentally, quantum state is conventionally referred to as quantum information (QI). In SQSS the initial owner partitions his/her secret QI among several recipients such that no recipient can retrieve the QI solely but the recipients entity can. SQSS has many important and potential applications in the future life. Consequently, after HBB's pioneering work much attention has been focused on this issue in both the theoretical and experimental aspects [4–20]. Now it has already become an important branch of QI field.

As we know, multi-particle entangled states as shared resources play crucial roles in quantum information processing including SQSS. Because of this, so far various entangled states such as Bell states [4, 12], GHZ states [21–23], graph states [24], cluster states [25, 26], genuine states [27, 28], have ever been explored as either quantum channels or their candidates. Recently W states have attracted many attentions [15, 29–34] due to their distinct advantages in quantum information and quantum computation [35]. As for as SQSS with W state is concerned, in 2006 Zheng [15] first proposed a three-party SQSS scheme by using an asymmetric three-qubit W state in the form of $|W_3\rangle = (\sqrt{2}|100\rangle + |010\rangle + |001\rangle)/2$. BTW, he simultaniously considered the problem of his scheme realization in ion-trap system, too. In 2007 Li and Qiu [30] presented their SQSS scheme by employing a quasi-symmetric arbitrary-m-qubit W state, i.e., $|W_m\rangle = \frac{1}{\sqrt{2(m-1)}}(\sqrt{m-1}|10\cdots00\rangle + |0\cdots010\rangle + \cdots + |00\cdots01\rangle)$. In 2008, by utilizing the $|W_3\rangle$ state as quantum channels, Zhang and Cheung [31] gave a novel idea for constituting a simple and economical three-party SQSS scheme suitable for some special ensembles of QI. Also in 2008, by using the $|W_3\rangle$ states Zuo et al. [32] and Pan et al. [33] generalized Zheng's work [15] and Zhang et al.'s work [31] to treat the two-qubit QI, respectively. Very recently, Liu et al. [36] proposed a four-party SQSS scheme by making use of an asymmetric four-qubit W state $|W_4\rangle = \frac{1}{2\sqrt{2}}(2|1000\rangle + \sqrt{2}|0100\rangle + |0010\rangle + |0001\rangle))$ as quantum channels. In this paper, we will propose a tripartite SQSS scheme by employing a different W state from those employed, specifically, we use a class of normalized asymmetric 3-qubit W state in the form of $|\mathcal{W}\rangle = a|001\rangle + b|010\rangle + \frac{1}{\sqrt{2}}|100\rangle$, where a and b are complex.

The organization of this paper is as follows. In Sect. 2, our scheme for bisplitting an arbitrary single-qubit QI is proposed in detail. Then in Sect. 3, the scheme is generalized to split an arbitrary N-qubit QI. Finally, a concise summary is given in Sect. 4.

2 Tripartite Scheme for Bisplitting Arbitrary Single-Qubit QI

In our scheme there are three legitimate users, say, Alice, Bob and Charlie. Alice is the sender while Bob and Charlie are the two receivers. Alice holds a secret QI in her qubit X. The QI can be expressed as

$$|\varphi\rangle_X = \alpha|0\rangle_X + \beta|1\rangle_X, \tag{1}$$

where α and β are complex and satisfied $|\alpha|^2 + |\beta|^2 = 1$. Alice wants to split her secret QI into two pieces and distribute them to Bob and Charlie securely via quantum channels by quantum mechanics method. The motivation of her such distribution is that, the original information can be retrieved if and only if the two receivers collaborate together, otherwise neither can get it successfully. In our scheme the quantum channel consists of an asymmetric 3-qubit W state, i.e.,

$$|\mathcal{W}\rangle_{ABC} = a|001\rangle_{ABC} + b|010\rangle_{ABC} + \frac{1}{\sqrt{2}}|100\rangle_{ABC}, \tag{2}$$

where $|a|^2 + |b|^2 = \frac{1}{2}$ and the qubits A, B and C belong to Alice, Bob and Charlie, respectively. In this condition, the joint state of the whole 4-qubit system reads

$$|\tau\rangle_{XABC} = |\varphi\rangle_X \otimes |\mathcal{W}\rangle_{ABC}. \tag{3}$$

As mentioned just, to split her QI by using quantum mechanics method via quantum channels, Alice first measures her two qubits X and A in the Bell-state bases $\{|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), |\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)\}$. Subsequently, Alice notifies Bob and Charlie of her measurement result. Incidentally, all the three legitimate users have already made an agreement in priori. Specifically, the two-cbit message "00", "01", "10" or "11" declared by Alice indicates her measurement outcome $|\psi^+\rangle_{XA}$, $|\psi^-\rangle_{XA}$, $|\phi^+\rangle_{XA}$ or $|\phi^-\rangle_{XA}$, respectively. The joint state of the 4-qubit system can be rewritten as

$$\begin{aligned} |\tau\rangle_{XABC} = \frac{1}{2}[&|\psi^+\rangle_{XA}|\kappa(\beta,\alpha)\rangle_{BC} + |\psi^-\rangle_{XA}|\kappa(\beta,-\alpha)\rangle_{BC} \\ &+ |\phi^+\rangle_{XA}|\kappa(\alpha,\beta)\rangle_{BC} + |\phi^-\rangle_{XA}|\kappa(\alpha,-\beta)\rangle_{BC}] \end{aligned} \tag{4}$$

with $|\kappa(x,y)\rangle \equiv x|00\rangle + y(\sqrt{2}a|01\rangle + \sqrt{2}b|10\rangle)$. Apparently, Alice's measurement induces one of four possible collapses of the state in the qubits B and C, i.e., $|\kappa(\beta,\alpha)\rangle_{BC}$, $|\kappa(\beta,-\alpha)\rangle_{BC}$, $\kappa(\alpha,\beta)\rangle_{BC}$ or $|\kappa(\alpha,-\beta)\rangle_{BC}$. All possible collapses occur with equal probability (1/4). Upon receiving Alice's message, Bob and Charlie then know the collapsed state in their qubits exactly. Without loss of generality and for the sake of easy expression, let us uniformly suppose the collapsed state is $|\kappa(x,y)\rangle \equiv x|00\rangle + y(\sqrt{2}a|01\rangle + \sqrt{2}b|10\rangle)$ at this stage. In this case, if Bob and Charlie cooperate together, they are readily to retrieve the QI as follows. Surely, the single-qubit QI split by Alice should inhabit a qubit conclusively. Because of this, Bob and Charlie need decide in whose qubit to restore the QI at first.

First, suppose the qubit C in Charlie's position is selected after Bob and Charlie's consultation. See Fig. 1(a) for illustration. After their decision, Bob and Charlie cooperate to perform the two-qubit unitary operation U on their qubits B and C, where the operation takes the following form under the basis vectors $\{|00\rangle_{BC}, |01\rangle_{BC}, |10\rangle_{BC}\}$,

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}a & \sqrt{2}b \\ 0 & \sqrt{2}b & -\sqrt{2}a \end{pmatrix}.$$

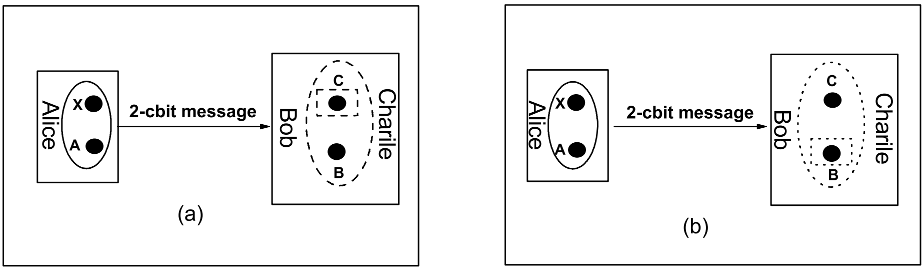

Fig. 1 Schematic demonstration of splitting an arbitrary single-qubit state with a class of asymmetric three-qubit W state. The *solid ellipse* denotes Alice's Bell-state measurement on her qubits X and A. The *dashed* and *dotted ellipses* represent Bob and Charlie's two-qubit unitary operations U and V on their qubits B and C. The *dashed* (*dotted*) *rectangle* characterizes Charlie's (Bob's) single-qubit operation on his qubit

Table 1 Alice's measurement outcome (AMO), Alice's two-cbit message (AM), the collapsed state (CS) of qubits B and C after Alice's measurement, the necessary unitary operation (NUO) and the final restored QI (RQI). $\sigma^y = |0\rangle\langle 1| - |1\rangle\langle 0|$, $\sigma^z = |0\rangle\langle 0| - |1\rangle\langle 1|$. See text for more details

AMO	AM	CS	NUO	RQI
$\|\psi^+\rangle_{XA}$	00	$\alpha(\sqrt{2}a\|01\rangle + \sqrt{2}b\|10\rangle)_{BC} + \beta\|00\rangle_{BC}$	$\sigma_C^x U_{BC}$	$\|\varphi\rangle_C$
$\|\psi^-\rangle_{XA}$	01	$\alpha(\sqrt{2}a\|01\rangle + \sqrt{2}b\|10\rangle)_{BC} - \beta\|00\rangle_{BC}$	$\sigma_C^y U_{BC}$	$\|\varphi\rangle_C$
$\|\phi^+\rangle_{XA}$	10	$\alpha\|00\rangle_{BC} + \beta(\sqrt{2}a\|01\rangle + \sqrt{2}b\|10\rangle)_{BC}$	U_{BC}	$\|\varphi\rangle_C$
$\|\phi^-\rangle_{XA}$	11	$\alpha\|00\rangle_{BC} - \beta(\sqrt{2}a\|01\rangle + \sqrt{2}b\|10\rangle)_{BC}$	$\sigma_C^z U_{BC}$	$\|\varphi\rangle_C$

Easily, one can see that, the operation converts the collapsed state $|\kappa(x,y)\rangle_{BC}$ into

$$U_{BC}|\kappa(x,y)\rangle_{BC} = |0\rangle_B(x|1\rangle + y|0\rangle)_C. \tag{5}$$

In terms of Alice's message, Charlie knows that he need further carry out an appropriate single-qubit operation on his qubit C to finally restore the QI. For instance, provided that Alice publicly announces "00", after Bob and Charlie's two-qubit operation U the state of the qubits B and C is $|0\rangle_B(\alpha|1\rangle + \beta|0\rangle)_C$. Therefore, Charlie only needs to perform $\sigma^x = |0\rangle\langle 1| + |1\rangle\langle 0|$ on his qubit C, that is, $\sigma_C^x|0\rangle_B(\alpha|1\rangle + \beta|0\rangle)_C = |0\rangle_B|\varphi\rangle_C$. Similarly, if Alice notices other message, Bob and Charlie can recover the QI by performing a proper unitary operation. Here we do not repeat them one by one anymore. The explicit correspondence relations among Alice's measurement outcome (AMO) on her qubits X and A, Alice's 2-cbit message (AM), the collapsed state (CS) in the qubits B and C after Alice's measurement, the necessary unitary operation (NUO) and the restored QI (RQI) in the assigned qubit, are summarized in the Table 1.

Secondly, suppose Bob and Charlie decide that the QI to be retrieved will finally inhabit the qubit B. See Fig. 1(b) for illustration. After their consultation, Bob and Charlie need to execute a 2-qubit unitary operation V on their qubits. The unitary operation V can be written under the basis vectors $\{|00\rangle_{BC}, |01\rangle_{BC}, |10\rangle_{BC}\}$ as,

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}b & \sqrt{2}a \\ 0 & \sqrt{2}a & -\sqrt{2}b \end{pmatrix}.$$

After this operation, the state of qubits B and C evolves to

$$V_{BC}|\kappa(x,y)\rangle_{BC} = (x|1\rangle + y|0\rangle)_B|0\rangle_C. \tag{6}$$

Easily, one can conclude that the QI can be restored in Bob's qubit B provided that he performs an appropriate single-qubit operation according to Alice's message. For instance, if Alice's published message is "00", to recover the QI Bob need perform a single-qubit operation σ^x on his qubit B. Similarly, if Alice's publishes other messages, Bob should perform other appropriate operations to retrieve the QI. See Table 2 for a brief summary.

Obviously, from Tables 1 and 2 one can see that either Bob or Charlie can retrieve the original QI with the other's help. This indicates that our scheme is symmetric in reconstructing the QI by either receiver. Furthermore, if the receivers are treated concretely, it is intuitive that our symmetric scheme can be viewed as a controlled teleportation one.

Table 2 Same as Table 1

AMO	AM	CS	NUO	RQI
$\vert\psi^+\rangle_{XA}$	00	$\alpha(\sqrt{2}a\vert 01\rangle+\sqrt{2}b\vert 10\rangle)_{BC}+\beta\vert 00\rangle_{BC}$	$\sigma_B^x V_{BC}$	$\vert\varphi\rangle_B$
$\vert\psi^-\rangle_{XA}$	01	$\alpha(\sqrt{2}a\vert 01\rangle+\sqrt{2}b\vert 10\rangle)_{BC}-\beta\vert 00\rangle_{BC}$	$\sigma_B^y V_{BC}$	$\vert\varphi\rangle_B$
$\vert\phi^+\rangle_{XA}$	10	$\alpha\vert 00\rangle_{BC}+\beta(\sqrt{2}a\vert 01\rangle+\sqrt{2}b\vert 10\rangle)_{BC}$	V_{BC}	$\vert\varphi\rangle_B$
$\vert\phi^-\rangle_{XA}$	11	$\alpha\vert 00\rangle_{BC}-\beta(\sqrt{2}a\vert 01\rangle+\sqrt{2}b\vert 10\rangle)_{BC}$	$\sigma_B^z V_{BC}$	$\vert\varphi\rangle_B$

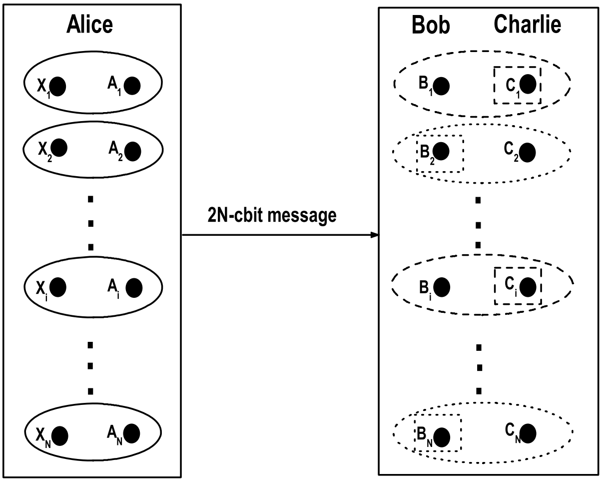

Fig. 2 Splitting an arbitrary N-qubit QI by utilizing N asymmetric W states. The signification of ellipses and rectangles are same to Fig. 1

3 Extension for Bisplitting Arbitrary N-Qubit QI

The above scheme can be easily generalized for partitioning an arbitrary N ($N \geq 2$)-qubit QI. The schematic demonstration is shown in Fig. 2. Suppose that the arbitrary QI Alice wants to split between Bob and Charlie is written as

$$|\Phi\rangle_{X_1X_2\cdots X_N} = \sum_{m_1=0}^{1}\sum_{m_2=0}^{1}\cdots\sum_{m_N=0}^{1} C_{m_1m_2\cdots m_N}|m_1m_2\cdots m_N\rangle_{X_1X_2\cdots X_N}, \tag{7}$$

where C's are complex and $|\Phi\rangle_{X_1X_2\cdots X_N}$ is normalized. Additionally, Alice, Bob and Charlie share in advance the following tensor product state of N three-qubit asymmetric W states as their quantum channels,

$$|\Theta\rangle_{ABC} = |\mathcal{W}\rangle_{A_1B_1C_1} \otimes |\mathcal{W}\rangle_{A_2B_2C_2} \otimes \cdots \otimes |\mathcal{W}\rangle_{A_NB_NC_N}. \tag{8}$$

Incidentally, the definition of $|\mathcal{W}\rangle_{A_iB_iC_i}$ $(i = 1, 2, \ldots, N)$ was given in (2) and the qubits $(A_1, A_2, \ldots, A_N)$ in Alice's site while $(B_1, B_2, \ldots, B_N)$ and $(C_1, C_2, \ldots, C_N)$ in Bob's and Charlie's positions, respectively. Hence, the total state of $4N$-qubit is

$$|\Gamma\rangle_{X_1X_2\cdots X_NA_1B_1C_1A_2B_2C_2\cdots A_NB_NC_N} = |\Phi\rangle_{X_1X_2\cdots X_N} \otimes |\Theta\rangle_{ABC}. \tag{9}$$

It can be rewritten as

$$|\Gamma\rangle_{X_1X_2\cdots X_NA_1B_1C_1A_2B_2C_2\cdots A_NB_NC_N} = \sum_{i_1=0}^{3}\sum_{i_2=0}^{3}\cdots\sum_{i_N=0}^{3}\prod_{j=1}^{N}|\varphi_{i_j}\rangle_{X_jA_j}|K_{i_1i_2\cdots i_N}\rangle_{B_1C_1B_2C_2\cdots B_NC_N}, \quad (10)$$

where $|\varphi_0\rangle$, $|\varphi_1\rangle$, $|\varphi_2\rangle$ and $|\varphi_3\rangle$ represent the four Bell states $|\psi^+\rangle$, $|\psi^-\rangle$, $|\phi^+\rangle$ and $|\phi^-\rangle$, respectively. To partition the QI $|\Phi\rangle_{X_1X_2\cdots X_N}$, Alice first performs N Bell-state measurements on her N qubit pairs (X_i, A_i) $(i = 1, 2, \ldots, N)$, respectively. Then she publishes a $2N$-cbit message via a classical channel. Surely, Alice's measurement will lead to the joint state collapse. From (10) one can see there are 4^N possible collapses and each appears with equal possibility. Once receiving Alice's message, Bob and Charlie then know the exact state their qubits have already collapsed to. We uniformly denote the collapsed state as $|K\rangle_{B_1C_1B_2C_2\cdots B_NC_N}$ hereafter. Similar to the case of splitting single-qubit QI, Bob and Charlie can recover the original QI successfully by performing first N two-qubit unitary operation $\Omega_{B_jC_j}$ $(j = 1, 2, \ldots, N)$ and then N single-qubit operations $\sigma^{i'}_{M_j}$ $(i' = 0, 1, 2, 3)$ to finally retrieve Alice's QI, i.e.,

$$\prod_{j=1}^{N}\sigma^{i'}_{M_j}\Omega_{B_jC_j}|K\rangle_{B_1C_1B_2C_2\cdots B_NC_N} = \left(\prod_{j=1}^{N}|0\rangle_{N_j}\right)|\Phi\rangle_{M_1M_2\cdots M_N}. \quad (11)$$

Note that here $\Omega_{B_jC_j}$ represents the unitary operation either $U_{B_jC_j}$ or $V_{B_jC_j}$ defined in Sect. 2, and M_j stands for the qubit C_j or B_j respectively. The switch between B and C determines the original QI can be recovered in qubits C_j or B_j. For examples, if the qubit C_j is assigned, $\Omega_{B_jC_j} = U_{B_jC_j}$; if B_j is chosen, then $\Omega_{B_jC_j} = V_{B_jC_j}$. In (11) the four single-qubit operations σ^0, σ^1, σ^2 and σ^3 express in turn Pauli matrices σ^x, I, σ^z and σ^y.

Now let us move to further elucidate our generalized scheme via a specific case, that is, bisplitting a two-qubit QI $|\Phi\rangle_{X_1X_2}$ corresponding to $j = 2$ in the general case. Suppose the outcomes of Alice's Bell-state measurements on her two qubit pairs (X_1, A_1) and (X_2, A_2) are $|\varphi_0\rangle_{X_1A_1}$ and $|\varphi_2\rangle_{X_2A_2}$, i.e., $|\psi^+\rangle_{X_1A_1}$ and $|\phi^+\rangle_{X_2A_2}$. In this case Alice first publicly announces a 4-cbit message "0010" via classical channel. With Alice's message, Bob and Charlie know that the state of their four qubits has collapsed to $|K_{02}\rangle_{B_1C_1B_2C_2}$. Assume Bob and Charlie cooperate together and decide the original QI is retrieved in qubits B_1 and C_2. In this situation, to accomplish the task, the two receivers perform the collective two-qubit unitary operation $U_{B_1C_1}V_{B_2C_2}$ first and then the single-qubit operation σ^x on qubit B_2. In this way, the original QI $|\Phi\rangle$ is conclusively restored in qubits B_2 and C_1, that is,

$$\sigma^x_{B_2}U_{B_1C_1}V_{B_2C_2}|K_{02}\rangle_{B_1C_1B_2C_2} = |00\rangle_{B_1C_2}|\Phi\rangle_{B_2C_1}. \quad (12)$$

Similarly, if Alice promulgates other messages, the original QI can be recovered successfully via Bob's and Charlie's appropriate unitary operations.

4 Summary

To summarize, in this paper we have proposed a tripartite scheme for bisplitting an arbitrary single-qubit QI via a class of asymmetric three-qubit W state. In our scheme, the sender measures her two particles in Bell-state bases and then declares the result. In terms of the sender's message, if the two receivers can restore the QI by performing an appropriate collective unitary operations if they cooperate together. We have also sketched its generalization for partitioning an arbitrary N-qubit QI.

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